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ABSTRACT

The California Assessment Program (CAP) administers tests to all public school students at certain grade levels, compiles the results, and provides information that allows educators to judge the effectiveness of their programs and make improvements. This sampler describes the types of assessment that CAP proposes to respond to the needed changes that reflect the recent curricular reforms in schools throughout California. The four types of assessment planned for CAP are: (1) open-ended problems; (2) enhanced multiple-choice questions; (3) investigations; and (4) portfolios. These modes of assessment are recommended for adaption for teachers of all grade levels and teachers are encouraged to use the examples in the booklet to enhance classroom instruction and to develop tasks for student assessment. After chapter 1 that describes the changes in assessment, the sampler is divided into six major parts: chapter 2, "Assessment of Mathematical Power"; chapter 3, "Types of Assessment"; chapter 4, "Performance Standards and Judging a Student's Work", chapter 5, "Implementation of Authentic Assessment in Your School," chapter 6, "Sample Problems." A "Participation and Feedback" page is given to ask participants' comments and suggestions about the sampler. (11 selected references) (MDH)

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A Sampler of Mathematics Assessment

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A Sampler of Mathematics Assessment

Prepared by

Tej Pandey

in Cooperation with the

Mathematics Assessment Development Team

and the

Mathematics Assessment Advisory Committee

Publishing Information

Working with both the Mathematics Assessment Development Team and the Mathematics Assessment Advisory Committee (see Acknowledgments on page v), Tej Pandey, Mathematics and Science Administrator in the California Assessment Program, developed *A Sampler of Mathematics Assessment*. The document was edited for publication by Theodore R. Smith, working in cooperation with Dr. Pandey, and it was prepared for photo-offset production by the staff of the Bureau of Publications. Cheryl Shawver McDonald designed and prepared the layout, and Steve Yee designed and prepared the cover.

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Copies of *A Sampler of Mathematics Assessment* are available for $4 each, plus sales tax for California purchasers, from the Bureau of Publications, Sales Unit, California Department of Education, P.O. Box 271, Sacramento, CA 95812-0271 (phone: 916-445-1260). A list of other publications available from the Department appears on page 55.

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A Vision of Mathematics Assessment in California

What if . . .

- **STUDENTS** eagerly anticipated assessment tasks as opportunities to produce something useful—a plan, a portfolio, a report, or results of an investigation?
- **TEACHERS** welcomed assessment as an integral part of the teaching/learning process that closely resembled actual instructional practice while fairly assessing their students' performance?
- **PARENTS** felt confident that their children were learning in order to become motivated and productive citizens who can function in a rapidly changing world?

The California Assessment Program and the teachers and educators of the Mathematics Assessment Development Team and the Mathematics Assessment Advisory Committee are dedicated to making this vision a reality. Please join us in accepting the challenge of creating these new forms of assessment.

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1

Changes in Assessment

Every year the California Assessment Program (CAP) administers tests to all public school students at certain grade levels, and then CAP's staff compiles the results for the schools, districts, counties, and the state as a whole. The purpose of CAP is to provide information that allows educators to judge the effectiveness of their programs and to give them the data they need to make improvements.

CAP has undergone substantial changes recently, and others are planned for the future. For example, changes have been proposed for some of the tests given currently in grades three, six, eight, and twelve; and a comprehensive student-level assessment has also been proposed. More importantly, the nature of the tests is changing to reflect new thinking and practices in teaching. Most significant, though, are profound changes in the way assessment of academic achievement is conceived; that is, what it means to know and then to be able to use that knowledge. Part of the focus of this booklet will be on that level of change as it applies to mathematics.

The focus of this booklet is on the assessment of what students know and are able to do in mathematics.

Currently, CAP is revising its mathematics assessment to reflect curricular reforms in schools throughout California. Teachers now place a greater emphasis than they did in the past on mathematical understandings, problem solving, hands-on learning experiences, collaborative work, and exposure to different strands of mathematics. In the wake of calls to strengthen mathematics instruction in California and the nation as a whole, more schools are certain to join the educational reform effort. This sampler describes the types of assessment that CAP is planning or proposing to use to support that reform.

At the core of mathematics reform is the idea of mathematical power. Mathematical power is developed as well as assessed by engaging students in worthwhile mathematical tasks—worthwhile both in the sense of being useful in a student's life and in the development of the student's academic abilities. Because the tasks that are being developed for the new assessments are grounded in sound educational

theory and shaped and tempered by the practice of classroom teachers who participate in creating the tasks, CAP is confident that the assessment will mirror and support good instruction.

The main purpose of *A Sampler of Mathematics Assessment* is to describe and illustrate the four types of assessment planned for CAP:

- Open-ended problems
- Enhanced multiple-choice questions
- Investigations
- Portfolios

These four types of assessment, which are described in detail in section 3 of this document, focus on mathematical understandings that students develop over a period of several years, rather than just one month or one school year. Thus, the assessment tasks described in this sampler can be used at or adapted to a number of grade levels, not merely the grade levels at which CAP administers tests. All teachers involved in any aspect of mathematical instruction have the responsibility to develop their students' mathematical power. Such teachers at all grade levels need to adapt these new modes of assessment for their classrooms in the same way that they adopt new modes of instruction.

Teachers of all grade levels need to adapt these new modes of assesssment for their classrooms in the same way that they adopt new modes of instruction.

CAP's new assessment will be phased in gradually, depending on the availability of resources. Open-ended problems have already been introduced at grade twelve. All the tests planned for the future will include enhanced multiple-choice questions and open-ended problems.

Investigations and portfolios will be implemented statewide after several years of pilot testing. In the first year of new assessment, investigations and portfolios will be administered in a statewide sample of schools. Each year the number of schools participating in assessments in which investigations and portfolios are used will be increased to accomplish full implementation in three to five years. Over the years, as the emphasis on open-ended problems, investigations, and portfolios increases, the emphasis on multiple-choice questions will decrease correspondingly.

A Sampler of Mathematics Assessment is intended for teachers of mathematics at both the elementary and secondary levels. Other educators who have responsibility for CAP, curriculum development, staff development, or other aspects of academic administration can also benefit from this sampler. Teachers are encouraged to use the examples in this

booklet to enhance classroom instruction and to develop tasks for student assessment.

This sampler is divided into six major parts. The next section, "Assessment of Mathematical Power," describes what mathematical power is and how it can be assessed. "Types of Assessment" includes samples of questions and students' responses. "Performance Standards and Judging a Student's Work" explains how students' responses can be judged against the preestablished performance standards. "Implementation of Authentic Assessment in Your School" provides background information that can help teachers and staff developers implement sound curricular and assessment practices. "Sample Problems" contains examples of open-ended and enhanced multiple-choice questions. Finally, a list of annotated references has been provided as an additional resource for the users of this document.

The sampler also includes a "Participation and Feedback" form at the end of the booklet. You are encouraged to use this form to communicate your concerns or suggestions to CAP and to volunteer for conducting pilot testing.

After trying out these new approaches to assessment, please send us your reactions by using the "Participation and Feedback" form at the end of the booklet.

Assessment of Mathematical Power

"What is tested is what gets taught. Tests must measure what is most important."

—Everybody Counts: A Report to the Nation on the Future of Mathematics Education

During the past several years, a growing consensus among mathematics educators is that current tests, usually multiple-choice questions that focus on narrow skills, do not assess the learning outcomes of "good" instruction. Multiple-choice assessment, in turn, has encouraged teaching that promotes learning facts and executing algorithms rather than developing mathematical understanding.

In addition to the shortcomings in tests, the weaknesses in the present curriculum are often acknowledged. A consensus among educators has emerged about what is essential in the teaching and learning of mathematics. On the state level this consensus is documented in the *Mathematics Framework for California Public Schools* (1985, draft 1991), the *Mathematics Model Curriculum Guide, K–8* (1987), and the *Model Curriculum Standards, Grades 9–12* (1985), published by the California Department of Education. On the national level this consensus is documented in *Curriculum and Evaluation Standards for School Mathematics* (1989), published by the National Council of Teachers of Mathematics, and *Everybody Counts* (1989) and *On the Shoulders of Giants* (1990), published by the National Research Council. These documents emphasize that developing mathematical power for *all* students is the goal of mathematics instruction.

This booklet does not describe in detail how students develop mathematical power. However, a short summary description of mathematical power can provide a background for examining the new assessments, for the goals of instruction must determine the structure of the new assessment.

Mathematical Power[1]

Mathematical power is the capacity to do purposeful and worthwhile mathematical work. The critical manifestation of mathematical power lies in the student's ability to employ:

- **Mathematical thinking**—to use knowledge and understanding to analyze, conjecture, design, evaluate, formulate, generalize, investigate, model, predict, transform, or verify
- **Mathematical understanding**—to use mathematical concepts and connections among concepts both within mathematics and across disciplines
- **Tools and techniques**—to efficiently and effectively solve mathematical problems; for example, using diagrams and tables, calculators and computers, manipulatives, and other concrete materials
- **Communication skills**—to communicate results with various audiences and for various purposes

Students develop mathematical power by working both individually and in groups on many types of problems and projects. Students should therefore be engaged in the breadth and depth of mathematics, rather than in repetition and drill. Mathematical activities should be based on real-life situations and explorations and should involve several mathematical ideas. The activities should present the student with the unexpected, have more than one "answer," and extend to other subject areas.

Such activities can stimulate and stretch students' mathematical thinking. By using a wide variety of mathematical manipulatives, tools, and other resources, students become involved and reflect on

When we give students the breadth and depth of mathematics, we provide activities that stretch across the seven strands of mathematics—number, measurement, geometry, logic, patterns and functions, statistics and probability, and algebra—and help them extend their mathematical work in to other subject areas.

[1]How do the four aspects contribute to mathematical power? An analogy with the generation of power in a gasoline engine may be helpful. Suppose that gasoline is like mathematical understanding, the process of combustion is like thinking, the engine housing corresponds to tools and techniques, and the transmission is analogous to communication skills. Then just as combustion in the engine converts fuel into energy, thinking serves as the spark to fuel mathematical knowledge to do purposeful work. Just as the engine housing and values facilitate combustion, tools and techniques allow us to think efficiently and effectively. And just as energy produced in the engine is channeled to the wheels by the transmission, the productive mathematical work is manifested only when students are able to communicate their ideas with the intended audience.

The capacity or the full extent of the work that the combustion produces and communicates through the transmission is revealed only when we put the engine in gear. So, too, are students able to show the extent of their mathematical power only when they are engaged in interesting and mathematically rich problems.

their activities. Students should be given the opportunity to express the results of their activities in a variety of oral and written products or presentations.

CAP's new assessments support this approach to instruction. By aligning assessment with instruction, teachers can use the assessment tasks to examine student understandings and to help develop instructional programs.

CAP's New Assessment Tasks

The Mathematics Assessment Advisory Committee and the Mathematics Assessment Development Team are planning and developing CAP assessment tasks. The CAP advisory committee is composed of researchers, curriculum experts, and teachers. The assessment development team is composed of classroom teachers, kindergarten through grade twelve. The team is divided into committees of ten to twelve teachers, organized by current CAP grade levels.

In order to assess mathematical power, the CAP committees have developed several types of assessment.

In order to assess mathematical power, the CAP committees have developed several types of assessment. As much as possible, each type requires students to:

- Restructure information rather than simply recall and reproduce it.
- Understand and use information in new and unfamiliar contexts.
- Explain *why* and *how* rather than just state a result of some arithmetic calculation or algebraic manipulation.
- Integrate and connect their conceptual understandings as they observe, experiment, reason, and interpret, as well as make decisions and draw conclusions in situations they encounter within and outside the school.
- Demonstrate persistence, imagination, and creativity, and show their own novel problem-solving approaches.

Furthermore, the committees used the characteristics given in Table 1 as a guide to determine the quality of assessment tasks.

TABLE 1

UNDESIRABLE AND DESIRABLE CHARACTERISTICS OF ASSESSMENT TASKS[1]

Characteristics		*Rationale for determining characteristic*
Undesirable	*Desirable*	
Contrived	Authentic	Processes used by students must be like those used by people who use math.
Tangential	Essential	Fits the core of the curriculum; i.e., hits the *big* ideas, not just an enrichment exercise.
Superficial	In-depth	Leads to other problems and questions. Ripe with possibilities. Leads to connections.
Uninteresting	Engaging	Thought provoking. Fosters persistence.
Won't Work	Feasible	Can be done easily and safely within the constraints of the school and classroom.
Single-dimensional	Multidimensional	Integrated with other topics and subjects.
Structured	Open	Allows multiple entry points, multiple points of view.
Audience and purpose implied or unclear	Audience and purpose specified	Appropriate diagrams, tables, charts, symbols, and written descriptions used to convince the reader.

[1]These guidelines were first developed by Grant Wiggins of CLASS. The CAP committees adapted and extended the guidelines for CAP's assessment development.

CAP's New Assessment Types

CAP is developing four types of assessments:

- Open-ended problems
- Enhanced multiple-choice questions
- Investigations
- Portfolios

These four types of assessment differ from each other in many ways, as summarized in Table 2. CAP will use all four types in order to assess as many dimensions of mathematical power as possible. The specific combination of the four assessment types will be determined by extensive piloting and field testing. This combination will evolve and change as new research data become available.

TABLE 2

FEATURES OF FOUR TYPES OF ASSESSMENT[1]

	Nature of feature, by type of assessment			
Feature	*Enhanced multiple-choice questions*	*Open-ended problems*	*Investigations*	*Portfolios*
Time per task	2-3 minutes	15-45 minutes	2-3 hours	2-3 weeks
Calculator	Yes	Yes	Yes	Yes
Manipulatives	No	When appropriate	Yes	Yes
Correct answer	One	Many	Many	Many
Who creates responses?	Item writer	Student	Student	Student
Scoring	Answer key	Judging	Judging	Judging
Model of good instruction	No	Yes	Yes	Yes
Collaboration	No	No	Yes	Yes
Student self-assessment	Little	Some	Much	Most
Usefulness to teacher	Some	Very much	Very much	Most

[1]**Notes to Table 2:**

Time per task. Under time pressure, students can answer only straightforward, narrowly-focused, or fact-related questions. Speed and memorization become more important than creative solutions and extensions, which show thinking but require more time.

Calculators. Calculators are as indispensable as paper or pencil. When students use calculators appropriately, they show evidence of mathematical power.

Manipulatives. Manipulatives and other resources help students clarify thinking and communicate with others.

Correct answer. In multiple-choice questions students are required to think as the item writer thought to find the single correct response. In open-ended questions and investigations, students can use their own experiences to produce one of many correct solutions.

Who creates responses? In multiple-choice questions, students choose responses. In open-ended problems and investigations, they are required to construct their own responses.

Scoring. Multiple-choice questions have a single correct answer. When students construct responses for open-ended problems or investigations, many correct and even creative solutions are possible. These responses are scored by experts—specifically, trained classroom teachers.

Model of good instruction. Assessment tasks, such as open-ended problems and investigations, can provide models and starting points for good instruction.

Collaboration. In the workplace and other life situations, people have to learn to work in groups as well as individually. Student collaboration develops cooperative skills and helps clarify thinking.

Student self-assessment. When students understand what they have accomplished and examine why they chose certain strategies, they are better able to continue their development. This self-assessment takes place when students compare their past and present work and have opportunities to see many examples of good work and compare their own work with those examples.

Usefulness to teacher. In contrast to multiple-choice questions, open-ended problems and investigations reveal what students can actually do and where their difficulties lie. This information can then be used by teachers to modify instruction according to students' learning needs.

3

Types of Assessment

Section 2 provided a general overview of the four types of assessments developed for CAP. This section describes specific features of each of the four types of assessments, with illustrative examples. A discussion of open-ended problems is presented first, followed by enhanced multiple-choice questions, investigations, and portfolios. Suggestions to teachers are also offered for developing student assessment tasks.

Open-Ended Problems

An open-ended problem presents students with a description of a problem situation, and it poses a question for students to respond to in writing. The question is designed to find how well the student can think, solve, and communicate about the given situation. The problem should be open-ended enough to allow for more than one solution (or path leading to the solution) without being vague.

Most open-ended problems developed by CAP are designed so that students can complete them in approximately 15 minutes. For some problems, however, students may use up to 45 minutes. Calculators and, in some cases, manipulatives may be needed. In particular, an open-ended question provides:

Teachers can write assessment tasks by combining particular content with these four characteristics: interesting situations, multiple entry points, multiple solutions, and an audience.

- **An interesting situation** that engages students and involves several mathematical concepts
- **Multiple entry points** that allow students at many levels of understanding to begin working on the problem
- **Multiple solutions** that allow students to make their own assumptions and develop creative responses
- **An audience** that creates the need for students to communicate effectively, using appropriate tools such as charts, graphs, and diagrams

Although different students may approach such problems in different ways, mathematically powerful students will:

- Find key mathematical elements of the problem situation.
- Explore mathematical relationships and formulate hypotheses.

- Demonstrate and explain their unique resolution of the problem through diagrams, graphs, charts, and other methods and extend their mathematical thinking to related problems or situations.
- Communicate their findings effectively to an audience.

While writing open-ended problems, CAP committees found it helpful to keep in perspective the essential requirements described above as well as the ways in which students might respond to the problems. However, the above lists are not a step-by-step procedure for creating or evaluating open-ended questions; each open-ended question contains these characteristics to varying degrees.

Examples of Open-Ended Problems

The following two examples with student responses illustrate the features of open-ended problems. Example 1 has a situation that the student can imagine and an audience that the student must "help." Example 2, like the first one, has a situation and requires the student to explain the solution to an audience. The student can use a variety of strategies to arrive at a solution.

Example 1—Bus Ride

A friend of yours, who just moved to the United States, must ride the bus to and from school each day. The bus ride costs 50 cents. Your friend must have exact change and must use only nickels, dimes, and quarters.

Your friend has a problem because she does not yet understand our money, and she does not know how to count our money.

Help your friend find the right coins to give to the bus driver. Draw and write something on a whole sheet of paper that can help her. She needs a sheet of paper that can show which combinations of coins can be used to pay for the 50-cent bus ride.

Be sure to organize your paper so it is clear and helpful for your friend.

The problem of the bus ride is open-ended because students must create their own diagrammatic way to show their solution to a particular situation and audience.

In responding to this problem, a mathematically powerful student would try to:

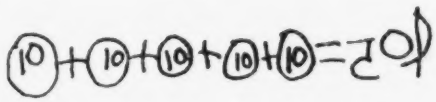
- Identify important elements of the problem, such as various denominations of U.S. coins.
- Explore possible combinations that yield 50 cents and determine that all possible combinations have been accounted for.

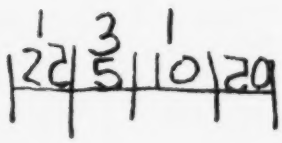
- Show combinations by using illustrations, tables, or charts.
- Present the information in a manner most helpful to the "friend."

The student:

- Shows some computational proficiency
- Creates only a few combinations
- Demonstrates a creative approach to represent combinations
- Does not communicate to the intended audience

Student Response 1

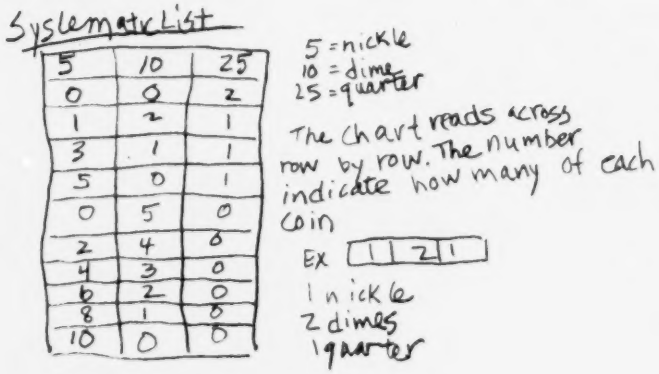




The student:

- Uses a chart systematically and effectively
- Partially communicates with intended audience
- Demonstrates computational proficiency

Student Response 2



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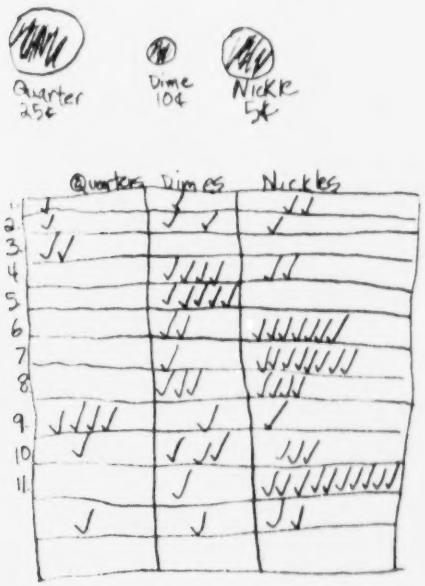
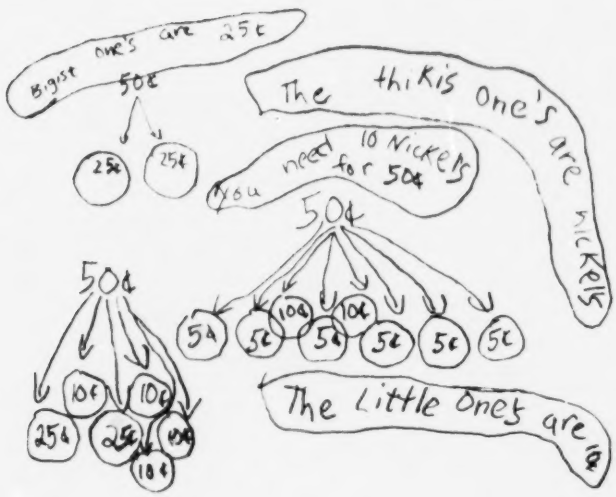
Student Response 3



The student:

- Uses a table effectively to show information
- Demonstrates computational proficiency
- Makes all combinations
- Makes a minor error in combination or computation
- Communicates pictorially to audience

Student Response 4



The student:

- Communicates with the audience
- Demonstrates some proficiency with combinations
- Uses both words and diagrams to show combinations

When this problem has been administered, students have arrived at over 60 correct interpretations or conclusions about the data. This diversity in the responses shows that the problem is open-ended; multiple solutions are possible and are even encouraged by the structure of the problem.

Example 2—Survey of Smoking

The journalism class of Exeter High School surveyed 100 of the school's 2,000 students about their smoking habits. The results of the survey follow:

38	**Never smoked**
11	**Current smoker who has smoked less than one year**
24	**Current smoker who has smoked more than one year**
18	**Quit smoking less than one year ago**
9	**Quit smoking more than one year ago**
100	**Total**

Write a short article that could appear in a school newspaper about the results of the survey. Include five statements that show interpretations or conclusions you derive from the survey data.

In responding to this problem, the student has an opportunity to demonstrate an understanding of the mathematical implications of the data and to interpret the meaning of that data. Although there is no step-by-step process that a student must follow, a mathematically powerful student would try to:

- Identify important elements of the problem, such as the different groups of students and how much they smoked.
- Explore the mathematical relationships within the problem by comparing the relative size of each group.
- Interpret the data.
- Present ideas in a manner consistent with the intended audience; e.g., high school students.

Student Response 1

In a recent poll taken at Exeter High School of 100 students chosen randomly, it was shown that about two-thirds (65%) of our classmates have never smoked or are no longer smoking. Unfortunately, 35% of those students polled have picked up this bad habbit. Of those students who do not smoke, 38% of them have never tried smoking, 18% have quit smoking within the last year and 9% of our former smokers have not had a puff in over a year.

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Student Response 1 (Continued)

Of that approximate third of the class that currently smokes, 24% have been smoking for over a year but we can still hope that the 11% who have only been smoking a short time (under a year) will come to their senses. It is sad to note that 62% of our students have tried smoking at one time.

Student Response 2

The journalism class of Exeter High School surveyed 100 of the schools 2,000 students about their smoking habits, and they came up with some startling evidence. Of the current smokers 67% of them have been smoking more than a year. And out of the number polled 62% had smoked at one time or another. And something else did rise up: of those 62 kids that have smoked, only 44% have quit. That really a shame. Another shame is that only 38% of the students polled had never smoked.

One thing that encourages me is that those who have quit smoking less than one year ago outnumber those who have started less than a year ago 18 to 11. It's time to educate our kids about the effect of cigarettes as well as drugs and alcohol.

Unlike the first problem, this problem allows the student to arrive at one of many possible correct solutions, depending on the assumptions he or she makes.

Several open-ended questions that are formatted for classroom use are included in the "Sample Problems" section at the end of this booklet. Although CAP will administer open-ended questions on an individual basis, any of the open-ended problems in this sampler can be given as classroom exercises to be completed individually or in small groups. These problems can also serve as the starting point for broader instruction and investigation.

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Enhanced Multiple-Choice Questions

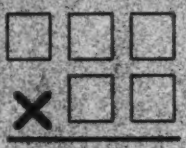
Traditional multiple-choice questions usually assess concepts in isolation, using routine procedures and facts. Students are given only 30–60 seconds to find the single correct answer.

Enhanced multiple-choice questions go beyond the assessment of a single skill. They require students to make connections among several concepts to arrive at the correct answer. They also allow students to use more than one mathematical strategy to solve the problem. Because of their relative complexity, enhanced multiple-choice questions require two to three minutes to answer.

When answering enhanced multiple-choice questions, students must use several types of conceptual understanding; merely recalling facts or procedures is not enough.

Two examples of enhanced multiple-choice questions are given below. The annotations explain how questions were written to capture a student's mathematical understanding.

Example 1—Digits



The five digits—1, 2, 3, 4, and 5—are placed in the boxes above to form a multiplication problem. If they are placed to give a maximum product, the product will fall between:

A. 10,000 and 22,000
B. 22,001 and 22,300
C. 22,301 and 22,400
D. 22,401 and 22,500

This problem deals with multiplication and place value. Students can approach the problem purely by trial and error or by trial and error in a systematic way. Teachers can use variations of this problem to provide rich instruction. For example, they can ask students to find the minimum product or a product that is closest to a given number. Alternately, the multiplication operation can be replaced by addition, subtraction, or division. The problem can be modified for use at grades three through twelve.

Example 2—Rolling Triangle

A piece of cardboard shaped like an equilateral triangle, with each side 5 cm in length, is rolled to the right a number of times. If the triangle stops so that the letter "T" is again in the upright position, which one of the following distances could it have rolled?

A. 24 cm
B. 30 cm
C. 60 cm
D. 90 cm

Feel free to use these problems in their current form or develop your own extensions.

In order to answer this question, the student needs to understand and use representation, spatial reasoning, and multiple pattern recognition. The problem can be solved algorithmically or by logical thinking. Again, teachers can develop variations of this problem for classroom use. For example, the triangle can be replaced with a square.

Additional examples of enhanced multiple-choice questions for use at various grade levels appear in the section of "Sample Problems." Classroom teachers can use these problems in their current form or develop extensions to meet their instructional and assessment needs.

Investigations

CAP investigations include all the features of open-ended problems. They present students with a situation, allow for multiple entry points and multiple paths to a solution, and provide opportunities to communicate to an audience. In addition during an investigation students:

- Use an extended period of time to explore as well as apply mathematical concepts (about three hours over two to three days).
- Collaborate with one or more partners.
- Decide how to use a variety of manipulatives and other resources.
- Can show persistence by how they identify a problem, develop a plan, and explain results.

Investigations are designed to meet essentially all the criteria of a desirable assessment task (see Table 2). They are richer tasks than

open-ended problems, allowing for up to three hours to complete; hence, investigations are able to tap aspects of learning not accessible by open-ended or multiple-choice questions. Investigations can, in fact, be used to assess almost all aspects of the students' mathematical power. In other words, they can reveal how well students employ mathematical thinking and their understanding of concepts, tools, and techniques to accomplish a purpose and communicate their results.

The example that follows illustrates students' responses to one investigation. In this investigation, a pair of eighth grade students collects data to design a fair and interesting game that shows their understanding of probability.

Investigations involve several related activities that progress from directed to least directed.

The investigation is divided into three activities. Activity 1 is self-directed to familiarize students with collecting data, discussing the problem, and then writing individual responses. Activity 1 has enough structure that almost all students—even those lacking similar experiences—can handle it. During this activity, teachers may guide students to set up their work and answer their questions.

Activity 2 of this investigation is relatively less structured and mathematically more involved, requiring students to make predictions, gather data, check for the accuracy of their predictions, analyze and interpret their findings, and write explanations. Although teachers may guide students during this activity, they are encouraged to let students persist and try alternatives for themselves to prepare for the important third activity.

Activity 3 is the most open-ended component of the investigation. Students are required to carry out the activity without any teacher's guidance. In this activity students apply concepts of probability in an open-ended manner to design a coin-tossing game that is fair and interesting. They have to collect data to check for fairness, write the rules for someone else to play the game, and explain why their game is interesting. This activity provides students an opportunity to demonstrate their mathematical understandings and insights, creativity, and ability to communicate. Although the two students cooperate on the investigation, they write their results individually. Note especially figures 1 and 2.

As this example illustrates, investigations are powerful instructional as well as assessment tools. The whole process models good instruction, and student assessment is an almost incidental by-product, useful for both teachers and students.

Example—Coin Toss (Grades Seven Through Twelve)

Figure 1. Page one of the student's booklet

STUDENT A

CAP Math Investigations

Today you and your partner will be working together on an investigation. You will have 90 minutes to complete all tasks. You must complete your own answer sheets, but please discuss all ideas with your partner.

The investigation will require
- doing an activity
- making and recording observations
- answering questions

You will be scored on
- how well you work together
- how you use words and diagrams to show what happens and why

Activity 1: One-coin Toss

You and your partner are to take turns tossing a coin. Each of you is to toss the coin 50 times. Every time a coin is tossed, you will write the result on your sheet. Remember, both of you must record all information.

A. Make a Prediction (guess)

Discuss your ideas with your partner, then write. What do you think the total number of heads will be? 50 the total number of tails? 50
Tell why you think so.

Because there are only 2 sides and there is a 50/50 Chance to get what you want.

STUDENT B

CAP Math Investigations

Today you and your partner will be working together on an investigation. You will have 90 minutes to complete all tasks. You must complete your own answer sheets, but please discuss all ideas with your partner.

After discussion, each student communicates correct reasoning about probability from slightly different viewpoints.

Activity 1: One-coin Toss

You and your partner are to take turns tossing a coin. Each of you is to toss the coin 50 times. Every time a coin is tossed, you will write the result on your sheet. Remember, both of you must record all information.

A. Make a Prediction (guess)

Discuss your ideas with your partner, then write. What do you think the total number of heads will be? 50 the total number of tails? 50
Tell why you think so.

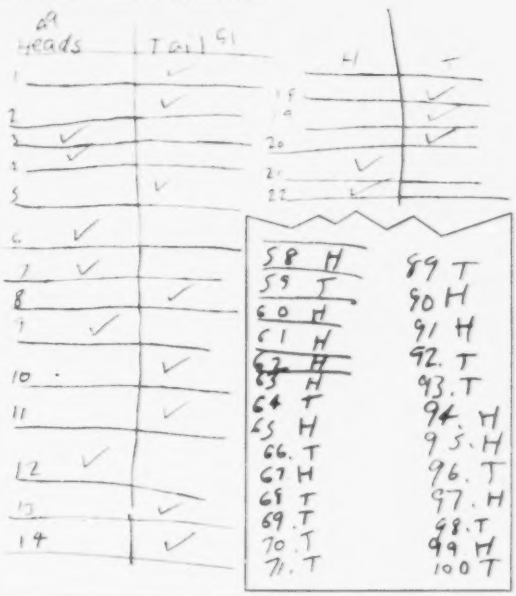
Because me and my partner are going to toss the coin 50 times (each person). 50+50=100 so, the chance to get heads and tails are ½. Therefore, you can get 50 for each side.

Figure 2. Page two of the student's booklet

STUDENT A

B. Do The Activity

Toss the coin and record the results below.



STUDENT B

B. Do The Activity

Toss the coin and record the results below.

In Taiwan, We use "正" to keep numbers.

Student work shows that both students can devise a table to record results. Student A does not use tallying; however, the student shows flexibility by changing the table in order to work more effectively (inset shows back side of page). Student B uses tallying method, and this shows the influence of past instruction.

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Figure 3. Page three of the student's booklet

STUDENT A	STUDENT B
Part C: Answer The Questions Now that you have finished the activity, answer the following questions. 1. Were the total number of heads and the total number of tails what you expected? yes Write and explain why you think the totals are the same or why you think the totals are different. I think that it is about the same as my guess because it is just 1 off. 2. If you tossed the coin 1000 times, how many times do you think you would get heads? 500 Explain why. because there is 50-50 chance to get each one so you would get around 500.	**Part C: Answer The Questions** Now that you have finished the activity, answer the following questions. 1. Were the total number of heads and the total number of tails what you expected? Yes Write and explain why you think the totals are the same or why you think the totals are different. I think the Totals are the same because it is really close to our result 2. If you tossed the coin 1000 times, how many times do you think you would get heads? 500 Explain why. Because 500 is $\frac{1}{2}$ of 1000 times.

Student work shows different reasoning. On question 2, Student A explains that the prediction of 500 is based on the understanding that "there is a 50-50 chance." Student B only states that 500 is one-half of 1000, but does not elaborate. On other questions, though, Student B does show understanding of chance.

Figure 4. Page four of the student's booklet

STUDENT A	STUDENT B
3. Imagine you are playing a game with the coin. a. If you get a point for every head that comes up and your partner gets a point for every tail that comes up, is this game fair? yes Explain why or why not. because there is a 50/50 Chance to get a side. b. If you get one point for every head that comes up, and your partner gets three points for every tail that comes up, is this game fair? no Explain why or why not. because still you get 50/50 chance but he gets more points if ~~he~~ gets the side than if you get ~~the~~ the side.	3. Imagine you are playing a game with the coin. a. If you get a point for every head that comes up and your partner gets a point for every tail that comes up, is this game fair? Yes Explain why or why not. Because the chance of two sides are equal. b. If you get one point for every head that comes up, and your partner gets three points for every tail that comes up, is this game fair? No Explain why or why not. Because the two sides get equal chance and probably your partner will get more point.

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Figure 5. Page five of the student's booklet

STUDENT A	STUDENT B
Activity 2: Two-coin Toss For this activity, you and your partner will toss identical coins. Keep track of which is coin A and which is coin B. Tossing the coins at the same time, do 100 tosses. **A. Make a Prediction** Before you toss the coins, figure out all combinations of heads and tails that are possible each time you toss the coins. Use the space below to figure out the combinations. H H H T T H T T What do you think the totals for each combination will be? Discuss your ideas and explain your thinking. 25 % chance, you are rolling 2 coins and each coin is a 50-50 chance	**Activity 2: Two-coin Toss** For this activity, you and your partner will toss identical coins. Keep track of which is coin A and which is coin B. Tossing the coins at the same time, do 100 tosses. **A. Make a Prediction** Before you toss the coins, figure out all combinations of heads and tails that are possible each time you toss the coins. Use the space below to figure out the combinations. A head + B head = $\frac{1}{4}$ = 25 times tail + tail = $\frac{1}{4}$ = 25 times head + tail = $\frac{1}{4}$ = 25 times tail + head = $\frac{1}{4}$ = 25 times What do you think the totals for each combination will be? Discuss your ideas and explain your thinking. 25, because there are 4 different chances. And they are equal. $100 \times \frac{1}{4} = 25$.

Here, Student B describes her reasoning effectively, whereas Student A does not clearly present his ideas.

Figure 6. Page six of the student's booklet

STUDENT A	STUDENT B
B. Do The Activity Start tossing the coins and record your results below. Remember, both of you should record all your results. A B: H H; A B: T H; A B: H T; A B: T T accidentally I Got 103 then I took off 3 by rolling 3 and taking those three off. So if equals 100.	**B. Do The Activity** Start tossing the coins and record your results below. Remember, both of you should record all your results. head + head | tail + tail | head + Tail tail + head We accidently toss the coin 103 times, and we decided to take 3 off But we don't remember what were the last three. So we roll three more times and cross off what we got.

Student work shows a clear chart and tallying method. Also the students collaboratively ("we") decided to use the concept of a random toss to correct an error.

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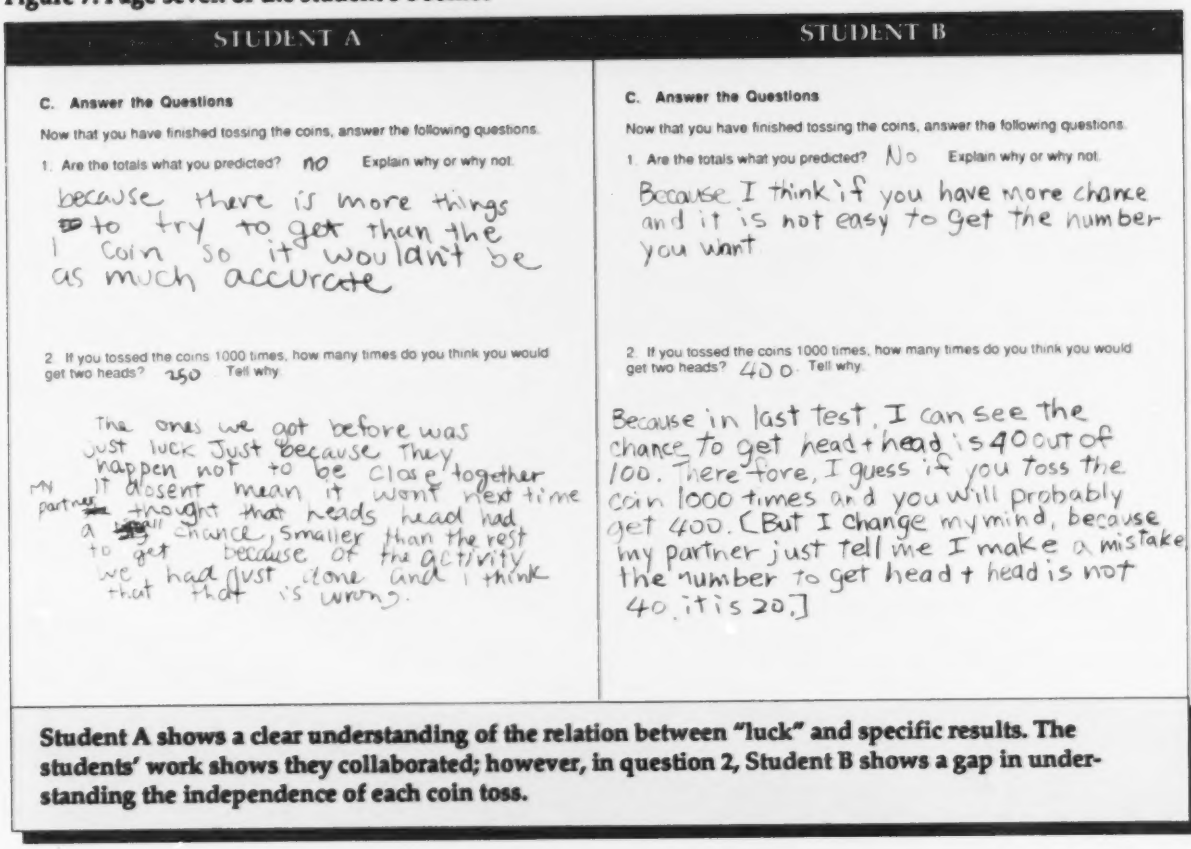
Figure 7. Page seven of the student's booklet



STUDENT A

C. **Answer the Questions**

Now that you have finished tossing the coins, answer the following questions.

1. Are the totals what you predicted? no Explain why or why not.

because there is more things to try to get than the 1 coin so it wouldn't be as much accurate

2. If you tossed the coins 1000 times, how many times do you think you would get two heads? 250 Tell why.

The ones we got before was just luck just because they happen not to be close together it dosent mean it wont next time my partner thought that heads head had a small chance, smaller than the rest to get because of the activity we had just done and I think that that is wrong.

STUDENT B

C. **Answer the Questions**

Now that you have finished tossing the coins, answer the following questions.

1. Are the totals what you predicted? No Explain why or why not.

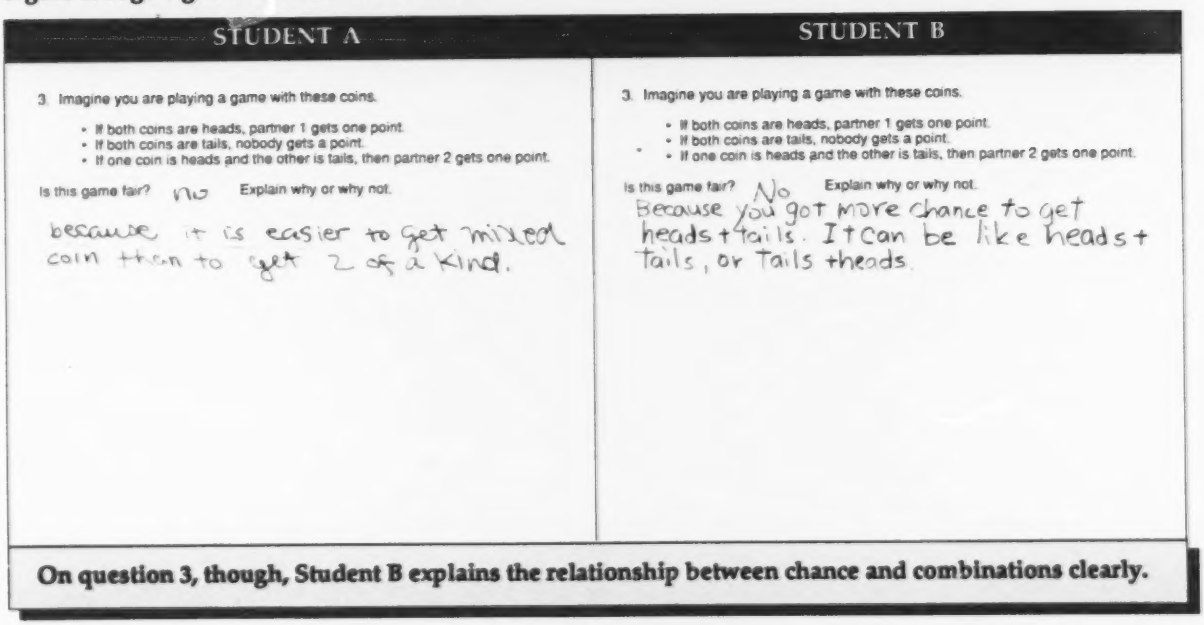
Because I think if you have more chance and it is not easy to get the number you want

2. If you tossed the coins 1000 times, how many times do you think you would get two heads? 400 Tell why.

Because in last test, I can see the chance to get head + head is 40 out of 100. Therefore, I guess if you toss the coin 1000 times and you will probably get 400. [But I change my mind, because my partner just tell me I make a mistake the number to get head + head is not 40, it is 20.]

Student A shows a clear understanding of the relation between "luck" and specific results. The students' work shows they collaborated; however, in question 2, Student B shows a gap in understanding the independence of each coin toss.

Figure 8. Page eight of the student's booklet



STUDENT A

3. Imagine you are playing a game with these coins.

- If both coins are heads, partner 1 gets one point.
- If both coins are tails, nobody gets a point.
- If one coin is heads and the other is tails, then partner 2 gets one point.

Is this game fair? no Explain why or why not.

because it is easier to get mixed coin than to get 2 of a kind.

STUDENT B

3. Imagine you are playing a game with these coins.

- If both coins are heads, partner 1 gets one point.
- If both coins are tails, nobody gets a point.
- If one coin is heads and the other is tails, then partner 2 gets one point.

Is this game fair? No Explain why or why not.

Because you got more chance to get heads + tails. It can be like heads + tails, or Tails + heads.

On question 3, though, Student B explains the relationship between chance and combinations clearly.

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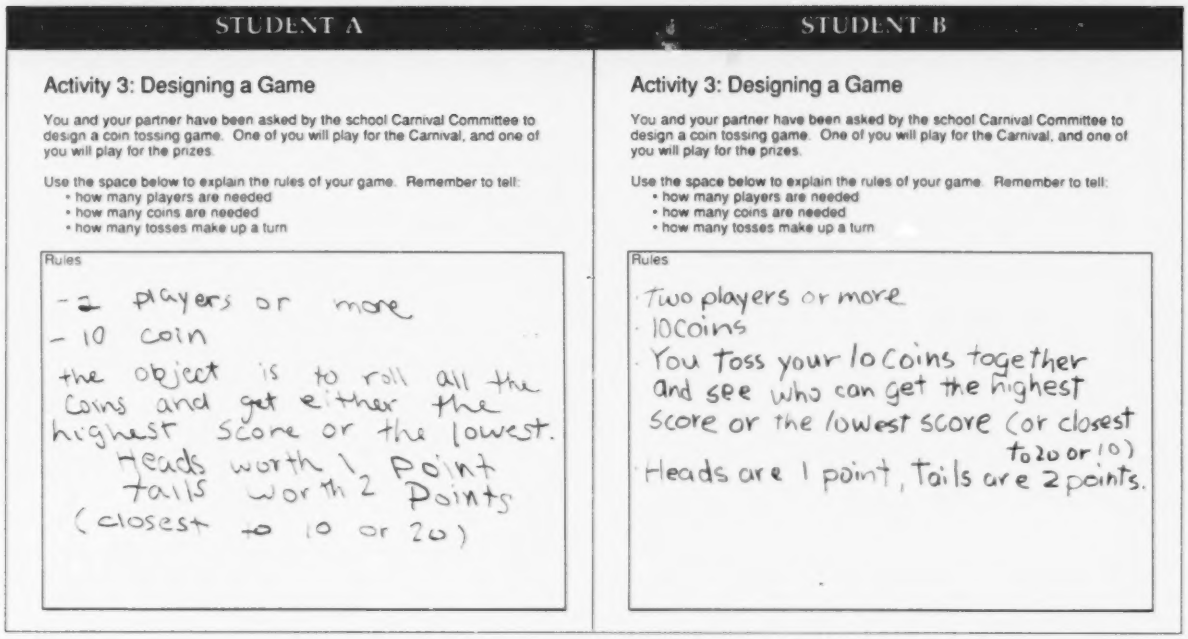
Figure 9. Page nine of the student's booklet



STUDENT A

Activity 3: Designing a Game

You and your partner have been asked by the school Carnival Committee to design a coin tossing game. One of you will play for the Carnival, and one of you will play for the prizes.

Use the space below to explain the rules of your game. Remember to tell:
- how many players are needed
- how many coins are needed
- how many tosses make up a turn

Rules

- 2 players or more
- 10 coin
the object is to roll all the coins and get either the highest score or the lowest.
Heads worth 1 point
tails worth 2 points
(closest to 10 or 20)

STUDENT B

Activity 3: Designing a Game

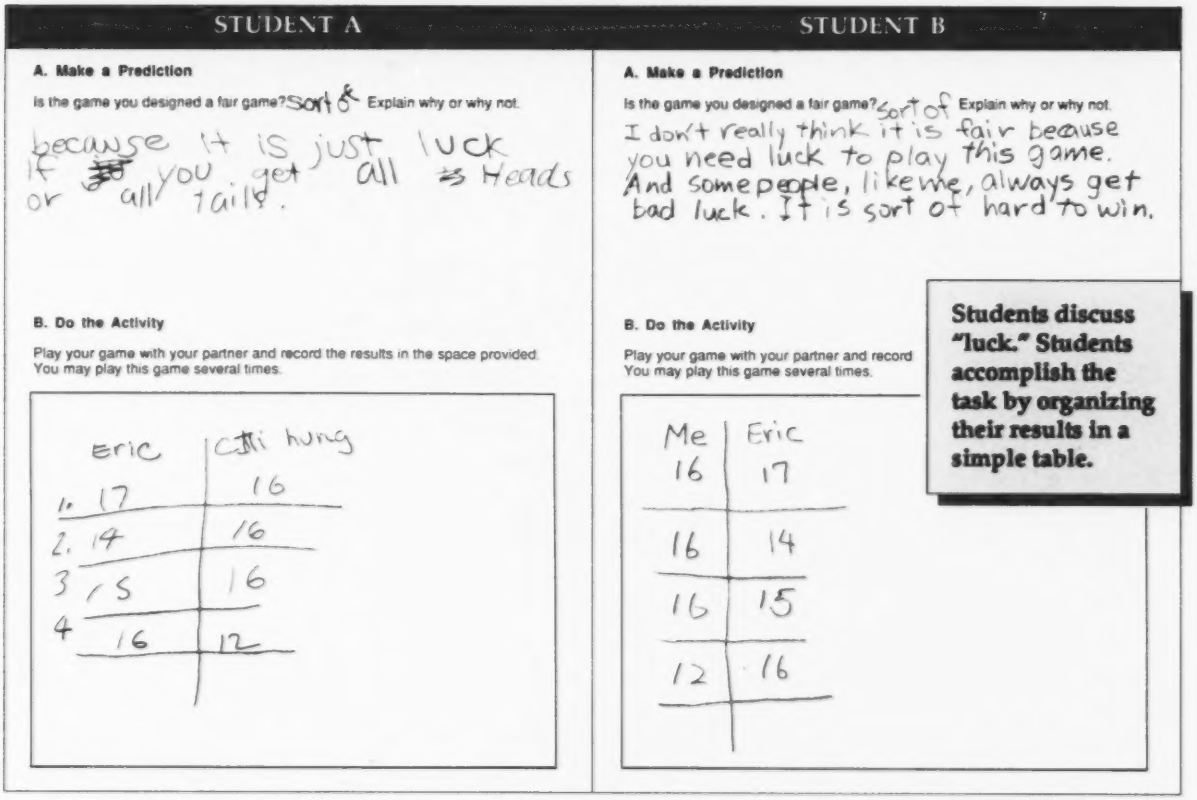
You and your partner have been asked by the school Carnival Committee to design a coin tossing game. One of you will play for the Carnival, and one of you will play for the prizes.

Use the space below to explain the rules of your game. Remember to tell:
- how many players are needed
- how many coins are needed
- how many tosses make up a turn

Rules

- Two players or more
- 10 coins
- You toss your 10 coins together and see who can get the highest score or the lowest score (or closest to 20 or 10)
- Heads are 1 point, Tails are 2 points.

Figure 10. Page ten of the student's booklet



STUDENT A

A. Make a Prediction

Is the game you designed a fair game? Sort of Explain why or why not.

because it is just luck if ~~you~~ you get all ~~5~~ Heads or all Tails.

B. Do the Activity

Play your game with your partner and record the results in the space provided. You may play this game several times.

	Eric	Chi hung
1.	17	16
2.	14	16
3	15	16
4	16	12

STUDENT B

A. Make a Prediction

Is the game you designed a fair game? sort of Explain why or why not.

I don't really think it is fair because you need luck to play this game. And some people, like me, always get bad luck. It is sort of hard to win.

B. Do the Activity

Play your game with your partner and record
You may play this game several times.

Me	Eric
16	17
16	14
16	15
12	16

Students discuss "luck." Students accomplish the task by organizing their results in a simple table.

Figure 11. Page eleven of the student's booklet

STUDENT A	STUDENT B
C. Answer the Questions Now that you have finished playing the game, answer the following questions. 1. Do you now think your game is fair? yes Explain why or why not. because the only ~~#~~ way you could win is with luck and it is only lucky if you get a high or low score. 2. If you played your game 1000 times, would the results be the same? maybe Explain why or why not. because sometimes you get lucky and sometimes you don't 3. Imagine that the Carnival is losing too many prizes. Change your game a little, so that the Carnival will not lose so many prizes. Describe these changes. You should add the number of coins and raise the number of people that should play at a time you should do that because if you add the coins, there is a bigger range and it is harder to get high or low. if you raise the number of people, more people will try to win and you get more competition.	**C. Answer the Questions** Now that you have finished playing the game, answer the following questions. 1. Do you now think your game is fair? Yes Explain why or why not. After playing, I realized I don't always get bad luck so I think it is fair. 2. If you played your game 1000 times, would the results be the same? maybe Explain why or why not. maybe not. Because it depents on your luck, and the results can't be the same. 3. Imagine that the Carnival is losing too many prizes. Change your game a little, so that the Carnival will not lose so many prizes. Describe these changes. We should add the number of coins and rise the players, so that the Carnival will not lose many prizes. We changed it because if you get more coins and it is hard to get close the biggest number or the smallest number (More harder) It is the same thing on the players.

Students show understanding of the link between the number of combinations or players and the number of prizes given.

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And it should be noted that teachers can vary or extend the coin-tossing game by having their students toss three coins, toss a die and a coin, or further analyze what *fair* means.

Portfolios

The portfolio is a new and powerful method of assessing students' mathematical learning. An assessment portfolio is a planned selection of students' work collected from the work done throughout the school year. Portfolios should include products that reflect a student's developing mathematical power. An assessment portfolio, however, is not the same as a student's folder. A student's folder may contain all of the student's work, whereas the portfolio contains a carefully chosen sample.

An assessment portfolio is a planned selection of students' work collected from the work done throughout the school year.

A portfolio should contain typical samples of work as well as the best efforts of the student. A "typical sample" should show evidence of growth in learning—to what degree the student has developed mathematical power. The typical sample should be selected by the teacher and should include work done toward the beginning of instruction as well as near the end of instruction. An "exhibition sample," on the other hand, should be selected by the student in consultation with the teacher. Exhibition samples should be accompanied by a written statement by the student to tell why he or she selected the samples. The exhibition pieces will indicate the result of the student's best effort and the level of perseverance.

The number of pieces in a portfolio can vary to meet the needs of the teacher and the instructional program. The following recommendations assume a total of five to ten pieces in the portfolio:

- **Individual work** (select four pieces). These are pieces of students' individual work on interesting and challenging mathematics problems. Student writing must accompany each work. The student could have worked on the problem as a member of a small group; however, the write-up of the problem should be individual. Two pieces should be selected from the beginning and two from near the end of an instructional period.
- **Group investigation report** (one piece). This should be a group-developed report of an investigation that took a minimum of three to five days to complete. The report should include the problem,

work reflecting progress of the student, difficulties encountered, and the conclusions or final products. (All members of the group should include the same report in their individual portfolios, with names of all students in the group listed.)

- **Individual reflective or imaginative pieces** (one or two). This is a piece of writing in which students reflect, imagine, and think over the work they have done in their mathematics class.
- **Other pieces** (one or two). This can be any other work that the teacher or student determines should be included in the portfolio.

These guidelines may be refined in future years. For example, the pieces may include teachers' observations or results of classroom assessments.

A properly designed assessment portfolio can serve four important purposes. It can allow:

- Teachers to assess the growth of students' understanding of mathematics.
- Students to keep a record of their achievement and progress (self-assessment).
- Teachers and parents to communicate about students' work.
- Teachers to collaborate with other teachers to reflect on their instructional program.

The portfolio concept of assessment is meant to remedy many of the concerns stemming from traditional standardized testing. Some of the important advantages of the portfolio are that it assesses learning in the context of the regular classroom environment, focuses on student work on complex tasks completed over a few weeks or a month, emphasizes mathematical thinking and performance rather than accuracy and speed of execution, and encourages students' self-assessment. Since portfolio assessment can cover almost all aspects of a student's mathematical power, it can provide the most authentic assessment of a student's performance.

The preceding description is intended to give an overview of portfolio assessment. Although a complete description of portfolio assessment design is beyond the scope of this document, the following information provides some direction in beginning a portfolio project:

- Obtain a copy of *Portfolio Assessment in Mathematics* by Judy Mumme (see Selected References).
- Talk to colleagues who have used portfolios.
- Explain to students in advance the rules for selecting portfolio pieces and how they will be judged.
- Impress on students that portfolio assessment will focus on their mathematical understanding through problem-solving approaches: drawing diagrams, giving examples, writing statements, and sharing their work as they complete projects.
- Use materials such as open-ended problems and investigations, which allow students to express their understanding, instead of using single-answer work sheets.
- Contact the project director of one of the California Mathematics Projects or the California Assessment Program for more assistance.

Portfolio assessment is still in development. In 1988-89, however, CAP conducted a pilot study of portfolio assessment involving 60 classroom teachers, and CAP is currently pilot testing several models of portfolio assessment. Extensive field testing will be conducted before portfolio assessment becomes an integral part of CAP.

Teachers interested in participating in CAP's pilot testing of this type of assessment should complete the "Participation and Feedback" form at the end of this booklet.

4

Performance Standards and Judging a Student's Work

This section describes a new way of looking at student achievement that CAP plans to adopt. A student's score on a test is commonly interpreted in relation to the score of other students or in relation to norms to determine the letter grade. Teachers using the new approach described here will evaluate a student's work by using a well-defined *standard of performance* rather than relating the work to that of other students or to a fixed (criterion) score on a test.

These well-defined performance standards will allow teachers to see strengths as well as deficiencies in student work and use these observations to help students develop mathematical power. Students will see what level of work is expected in order to achieve a higher level of performance. Just as significantly, students will view themselves as having control over the level of their accomplishments. This view should lead students to believe that attaining higher levels of performance depends not on innate abilities but on their own effort and the amount of time they spend on the tasks.

Describing Performance Standards

Performance standards describe qualitative differences in students' approaches to solving problems.

Performance standards are bench-mark descriptions of the quality of performance against which actual student work can be compared. The performance standards provide a basis for trained teachers to make judgments about the level of accomplishment demonstrated by student work.

Performance standards can be stated at several levels of specificity. The most *generic* statement, in Table 4, refers to all types of assessment and tasks. For example, the generic statement can be applied to an open-ended problem or to an investigation. To facilitate the use of performance standards at various grade levels and for different kinds of tasks, a teacher can expand the generic statement to provide *elaborated* descriptions. *Task-specific* statements, such as the descriptions in Table 5, refer to the standards for judging a student's performance on a

specific task. The specific statements are based directly on the generic performance standards, and they are developed through a process involving many teachers' judgments and a large sample of student work.

CAP's experience with assessment in direct writing and open-ended questions in mathematics has led CAP to distinguish six levels of performance, which are identified in Table 3. Also, setting the quality of student work necessary at each performance level must be consistent with what constitutes the mathematical power. As described in Section 2, mathematical power involves mathematical thinking, mathematical understanding, using tools and techniques, and communicating to achieve a purpose.

Teachers should consider how well the student has achieved the purpose of the task.

Examining to what degree the student has achieved the purpose of the task uncovers the degree of the student's mathematical power. While the student's grasp of the purpose of the task necessarily involves mathematical understanding and communication, the student's work related to these two aspects of mathematical power will also be examined in detail while judging his or her work. Each of the three aspects—achieving purpose, understanding, and communicating—can be articulated at six levels, with level 1 indicating the lowest quality of performance and level 6 indicating the highest, as shown in tables 3, 4, and 5.

Table 4, which is an outline of the performance standards for a student's work, was composed by elaborating for each of six perfor-

TABLE 3
SIX PERFORMANCE LEVELS, BY TYPE OF PERFORMANCE

Type of performance	*Level of performance*					
	6	5	4	3	2	1
Achieving purpose	**Goes beyond**	**All**	**Substantial**	**Partial**	**Minimal**	**None**
Understanding	In-depth	Thorough	Satisfactory	Gaps	Fragmented	Little
Communicating	Clear dynamic	Effective	Successful	Limited	Attempted	Unrelated

TABLE 4

PERFORMANCE STANDARDS FOR STUDENT WORK

Level	*Standard to be achieved for performance at specified level*
6	**Fully achieves the purpose of the task, while insightfully interpreting, extending beyond the task, or raising provocative questions.** Demonstrates an in-depth understanding of concepts and content. Communicates effectively and clearly to various audiences, using dynamic and diverse means.
5	**Accomplishes the purposes of the task.** Shows clear understanding of concepts. Communicates effectively.
4	**Substantially completes purposes of the task.** Displays understanding of major concepts, even though some less important ideas may be missing. Communicates successfully.
3	**Purpose of the task not fully achieved; needs elaboration; some strategies may be ineffectual or not appropriate; assumptions about the purposes may be flawed.** Gaps in conceptual understanding are evident. Limits communication to some important ideas; results may be incomplete or not clearly presented.
2	**Important purposes of the task not achieved; work may need redirection; approach to task may lead away from its completion.** Presents fragmented understanding of concepts; results may be incomplete or arguments may be weak. Attempts communication.
1	**Purposes of the task not accomplished.** Shows little evidence of appropriate reasoning. Does not successfully communicate relevant ideas; presents extraneous information.

mance levels the three types of performance in which a student can achieve: purpose of a task, mathematical understanding, and mathematical communication. In the statements that appear in Table 4, it must be emphasized that the overriding consideration is achieving the purpose of the tasks, rather than *elements* of quality of performance at each level.

The statements of the standards of performance in Table 4 can be applied at any grade level. However, it does not mean that a level 5 performance for a typical sixth grade student is the same as level 5 work by a typical eighth grade student. Naturally, on the same task, much more is expected of an eighth grade student than from a sixth grade student.

Another publication of the Department of Education, which is in the developmental stage, will contain extended performance standards, including examples of actual student work. These examples of student work will serve to illustrate each of the six levels of performance standards on a variety of tasks at key grade levels. CAP invites teachers at all grade levels to use the Participation and Feedback form near the end of this booklet to contribute examples of student work on tasks similar to those described in this sampler.

In the future, CAP's reports for each school will provide the percent of students who perform at each of the six levels of the performance standards. Teachers will then be able to see what percent of their students moved, for example, from level 4 in one year to level 5 in the subsequent year. Teachers will also be able to see what differentiates level 4 work from level 5 work based on actual student samples. Moreover, students will be able to see what it takes for them to perform at level 5. In essence, performance standards provide a link among school curriculum, instruction, and assessment.

Through instruction, teachers can influence how students approach problems and, thus, help students improve their performance. Students should never be labeled or limited because of a particular performance.

Judging a Student's Work

Comparing a student's work against the performance standards requires judging by trained teachers. Based on CAP's experience with the scoring of open-ended problems, the holistic judgment process of scoring a student's work has been found most useful. In holistic judgment, the judge assesses the overall quality of the response and the level of thinking demonstrated. This process contrasts with the analytic method, in which points are awarded for each step the student

gets "right," or subtracted for each step the student gets "wrong." Such a method does not allow for the scoring of a student's responses that do not follow the steps prescribed for analytic scoring. Holistic evaluation, on the other hand, is concerned with the overall mathematical thinking and understanding while downplaying the role of certain prespecified key points.

Holistic evaluation is concerned with the overall mathematical thinking and understanding while downplaying the role of certain prespecified key points.

The consistency or reliability of the judging process is considerably improved if judges have a more detailed guide for evaluating a student's work on a particular task than the generalized performance standards given in Table 4. This type of guide is called a task rubric.

A task rubric has the same six levels as the performance standards. It is, of course, assumed that tasks are open-ended and meet the criteria shown in Table 1. The description at each level, however, is specific to the task. The task rubric is developed by looking at a large number of student responses and first determining, by consensus, which responses correspond to each level of performance. After the consensus is validated, a description of the quality of student work is developed at each of the six levels, shown in Table 5.

The task rubric in Table 5 was developed for an open-ended problem given to twelfth grade students as part of the California Assessment Program. Students were allowed approximately 12 minutes to write their responses. This task is shown on page 14 with examples of student work on page 15.

Judging Students' Work to Improve Communication

Communication among professionals is the most popular mode of keeping abreast of the developments in a discipline; and researchers in medicine, law, and education communicate through publications and formal and informal meetings. CAP's experience with open-ended problems has demonstrated that judging students' work on rich, multidimensional problems encourages teacher-to-teacher interaction. Such interaction increases understanding of the way students approach problems and provides for a shared understanding of curriculum and instructional practices.

The new assessment encourages teacher-to-teacher collaboration for judging a student's performance.

Students can also create task rubrics and judge open-ended problems in a classroom setting. This activity promotes self-evaluation and provides yet another means for students to gain more knowledge and understanding of the content at hand.

TABLE 5
SMOKING SURVEY TASK RUBRIC

Level	*Description of work, by each performance level*
6	The student states five conclusions or interpretations, several of which evidence insightful comparison or synthesis, predict trends, discuss sampling techniques, demonstrate thinking about other issues for research, or in some way offer provocative questions. The response reflects analysis of the data and reveals unusual insight and variety of dimensions. Observations or interpretations are presented effectively either in a list of statements or in the format of an article.
5	The student demonstrates various dimensions of thought in completing the task of giving five conclusions or interpretations of the data. For example, stating that 29 percent (11 plus 18) of those surveyed made a decision about smoking within the last year is a different dimension from reporting that 38 percent had never smoked, or even adding 18 to 9 to get the fact that 27 percent had quit smoking. The student understands that extrapolation from the sample to the total student population mandates addressing sample reliability issues. Conclusions and interpretations are expressed effectively in either a list of statements or an article.
4	The student gives five conclusions or interpretations which are correct in concept but may have minor errors. The student understands the major implications of the survey and recognizes the possibility of bias in the sample. The explanation is successful, but it may lack detail.
3	The student gives an incomplete or superficial list of conclusions or interpretations, one or more of which may have major errors. For example, the response extrapolates to the entire student body without qualification. The conclusions or interpretations may be derived from the same line of reasoning. For example, changing each of the five numbers to percents of those surveyed or of the student body would give five conclusions from the same dimension or line of thought. The results are, on a whole, given coherently.

TABLE 5 (CONTINUED)

Level	*Description of work, by each performance level*
2	The student attempts to interpret or draw conclusions from the data but makes major conceptual errors or omissions. The response may make no reference to either the sample set or an extrapolation to the whole student body. For example, the student may simply state that 38 percent had never smoked. The student attempts to communicate, but the statements are unclear or fragmented.
1	The student copies the data or attempts to restate information given in the problem. No conclusions or interpretations are attempted, and the response reflects no understanding of the mathematical concepts. Any communication attempted is muddled, irrelevant, or superfluous.

5

Implementation of Authentic Assessment in Your School

What can you do to make assessment happen?

At a faculty meeting or in-service training session, you and your colleagues can discuss how effective the sample problems are in:

- Covering the broad mathematical understanding you believe is important. (If you feel something is missing, please write new problems and send them to CAP.)
- Eliciting students' mathematical understanding and enabling them to show the depth of their knowledge, background, and experience.
- Assessing dimensions of mathematical power.

You can also take one particular problem and discuss how the problem can be:

- Adapted for different grade levels.
- Modified to emphasize different aspects of the same situation; e.g., what additional questions could be added to focus students' work on other important aspects of the mathematical situation?
- Restructured to involve collaborative rather than individual work.
- Extended to include other related activities, so that the problem could become an investigation spanning several days' work, and resulting in a more elaborate product, such as a poster or booklet.

Ask those who provide and support your in-service training for opportunities to **collaborate** and **reflect on:**

- ✓ Mathematics programs that:
  - Emphasize exploring, **constructing meanings or understandings**, and communicating.
  - Include varied **instructional strategies**, particularly techniques in asking thought-provoking questions and processing/summarizing activities.

☑ How to:

- Integrate **manipulatives** in your programs as models for conceptual understanding and tools for problem solving.
- Understand the role of **calculators** in instruction and assessment.
- Use **collaborative groups** to analyze and solve problems, communicate their understanding, and share multiple solutions.

☑ Ways of evaluating:

- Student understanding by using varied **authentic assessment** tools.
- By embedding **self-assessment** in instruction.
- Student growth through the use of **mathematics portfolios** and **investigations**.

We suggest that you get together with colleagues at a faculty meeting or in-service training session and discuss the suggestions in this *Sampler of Mathematics Assessment*, with the above possibilities in mind. You and your colleagues can try out some of the items (or some of your adaptations or creations) in your classrooms. You can then share your students' responses and your experiences with other interested faculty members in order to start a new cycle of developing your curriculum and assessment.

When meeting with colleagues, get as many different perspectives on your developmental work as you can. You may want to structure discussion groups by grade level to encourage the sharing of common teaching experiences, or you may want to structure discussion groups across grade levels to encourage a more general discussion of issues.

Where can you find help?

For additional information and assistance, contact these resources:

☑ Mathematics Conferences

- California Mathematics Council's annual sectional conferences: central, north, and south
- Mathematics conferences of local affiliates (Contact your county office.)

- Regional conferences of the National Council of the Teachers of Mathematics

☑ County Offices

- Mathematics coordinators
- Mathematics workshop series (e.g., Math A)

☑ Organizations

- California Mathematics Projects
- Staff development programs, such as EQUALS
- State-developed replacement unit workshops (e.g., fifth-grade fractions project)
- CAP's scoring workshops and pilot projects

☑ Publications (See Selected References, pages 53-54.)

Sample Problems

This section is composed of a collection of both open-ended problems and enhanced multiple-choice questions that can be adapted for the appropriate grade level. Feel free to use any of these problems in your classroom.

Open-Ended Problems

Example 1

You may want to read the problems aloud to your students.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Scientists have decided to beam this 99 chart out to all known stars and galaxies. They hope that space aliens will pick up and understand something about our world. Some of the scientists thought it would be a good idea to send patterns that can be found on the 99 chart. If someone gets the patterns and the 99 chart, maybe they will learn more about us.

The scientists want your help. Find a pattern that you would like to send into space. You can show any pattern that you think will help the space aliens learn something about us or our number system.

Be sure to:

- Circle the numbers in your pattern on the 99 chart.
- Explain with words and pictures how you know that the numbers you circled show a pattern. (You may show your pattern in another way than on the 99 chart—on another piece of paper.)
- Explain why you chose your pattern and what you hope the space aliens will learn.

Example 2

A family of six ordered 13 hot dogs. Nine hot dogs had mustard, 3 had catsup, 8 had relish, 4 had both mustard and relish, and 3 had mustard, catsup, and relish. Explain to the clerk how many hot dogs had no relish or catsup. You may use drawings in your explanation.

Example 3

Imagine you are at a meeting of the Ice Cream Sales committee, which must decide which kind(s) of ice cream and how much to sell at the next sale. Here is some information from the last ice cream sale:

Flavor	*Number of bars ordered*	*Number of bars sold*
Chocolate	375	284
Vanilla	125	119
Strawberry	250	203
Orange	100	74
Lemon	100	56
Cherry	50	50

Someone on the committee says, "Most of the people like chocolate, so let's sell **only** chocolate next time."

Decide whether or not you agree with the above statement. Justify your opinion. If you disagree, devise and justify your own plan for the next ice cream sale.

- You may write and draw your response.
- If you would like to know other information about the ice cream sale, write down your questions.

Example 4

Your P.T.A. has decided to raise money by selling T-shirts with a beautiful geometric design. You have been asked to give your idea for a T-shirt design.

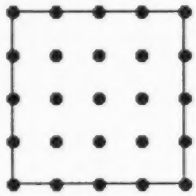
Draw a pattern for T-shirt designs. You may use any drawing tool to make your design. For your design, explain:

- What is geometric about the design.
- Why you think your design is mathematically beautiful.

Example 5

A visitor from outer space has just arrived. It is confused about our number system. It asks you, "Is 5 + 29 equal to 529?" Answer the visitor's question and explain your answer.

Example 6



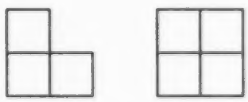
For the figure above, show $^1/_2$ in as many ways as you can. You may draw more figures, if necessary. For each way you find, explain how you know you have $^1/_2$.

Example 7

The cycle for the traffic light on Main Street is green for 55 seconds, yellow for 5 seconds, and red for 30 seconds. What is the probability of having to stop at the traffic light? Explain your reasoning and include consideration of how drivers react to yellow lights.

Example 8

Explain all you can about the similarities and differences between these two figures.

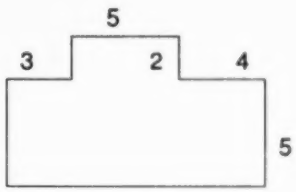


Your cousin is just beginning to learn more about numbers. She doesn't understand what $4 \times 3 = 12$ means. How would you explain this to her? You may use pictures or graphs.

Example 9

Create three figures with different shapes that have the same area as the given figure. Explain how you know your drawings have the same area as the original figure.

Example 10



The figure below represents $^1/_4$ of a whole shape. Create three possible whole shapes. Justify how you know that each new shape is one whole.

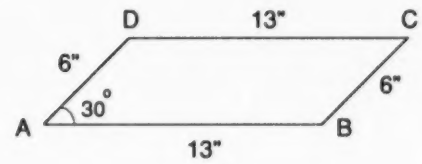
Example 11

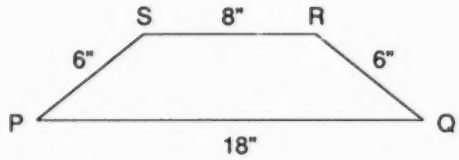
Bob says that an increase from 10 to 40 is 300% and, therefore, a decrease from 40 to 10 must be a 300% decrease. Do you agree with Bob? Explain your reasoning.

Example 12

Example 13

Give as many similarities and differences about these two figures as you can.





Example 14

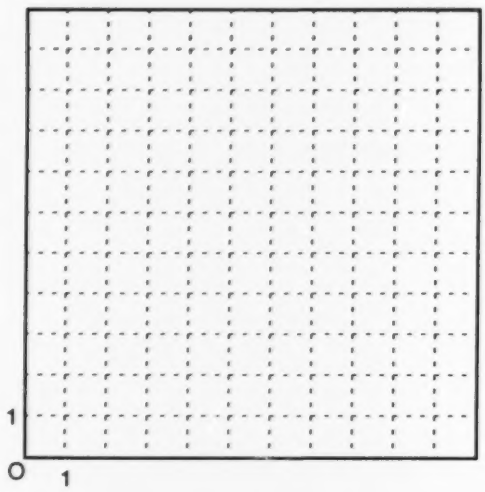
Henry wanted to make a box from an 8″ × 14″ piece of cardboard. He plans to cut squares from each corner and tape the corners. What are the dimensions of the box which will contain the largest volume? Explain to him how to find the dimension of the squares that he will need to cut from the cardboard in order to make the box.

Example 15

You plan to build a pen with at least one gate for your dog. Fencing is $1 per foot, fence posts are $2 each, and gates are $5 each. You can afford only $40 total. How would you design your pen in order to get the greatest possible area? Explain what you considered in making your plan.

Example 16

Imagine that you live on a flat world. The only way to move is through the coordinate plane. You are on a mission to capture a dragon that is threatening your village. If you can find out exactly where it is, you will be able to capture it. Other people in your village have already found out that the dragon does not lie farther north or east than (8, 6). Other people who have seen the dragon have figured out that it is 4 units long and $1^1/_2$ units wide.



Make up a short story that tells how you looked for the dragon by exploring the coordinate plane. Tell where you think the dragon was found.

Example 17

Imagine that you are working with a team of scientists. You are comparing the movement of ants and frogs. One of the scientists says, "If we observe an ant and a frog by themselves, each for five minutes, the frog will always travel farther."

Decide whether you agree with the above statement.

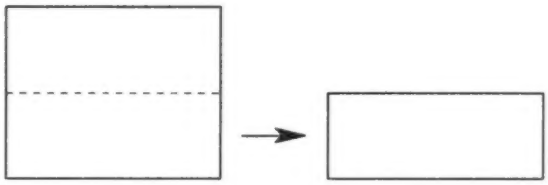
- Draw a diagram that shows the ant's movements for five minutes.
- Draw a diagram that shows the frog's movements for five minutes.
- Show all measurements.
- Use words and pictures to convince the other scientists that your opinion is correct.

Enhanced Multiple-Choice Questions

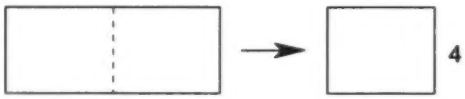
Example 1

In the classroom, you may want to ask students to justify their answers with writing and drawings.

Kim folded the rug in half.



Then she folded it again.

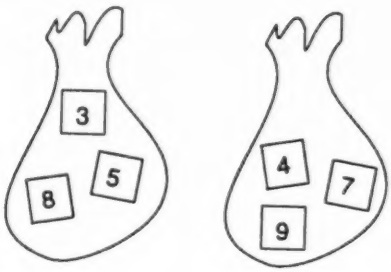


How big was the rug when it was opened?

A. 4×5
B. 5×8
C. 4×10
D. 8×10

Example 2

The bags can have equal sums by moving one number tile from each bag to the other bag. Which tiles should be moved?



A. 8 7

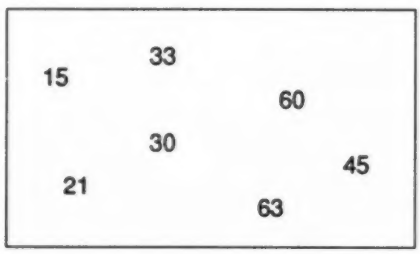
B. 3 9

C. 8 4

D. 5 7

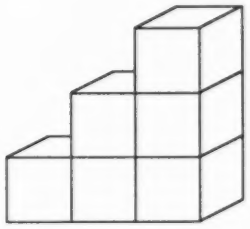
Example 3

What do all of these numbers have in common?



A. They are all even.
B. They are all odd.
C. They are multiples of 3.
D. They are all larger than 20.

Example 4



You are building a staircase out of cubes.

1 step = 1 cube
2 steps = 3 cubes
3 steps = 6 cubes

How many cubes does it takes to build a staircase that is 6 steps high?

A. 36 cubes
B. 28 cubes
C. 21 cubes
D. 15 cubes

Example 5

If the length and width of a rectangle are each doubled, the area is:

A. The same.
B. Two times as great.
C. Four times as great.
D. Eight times as great.

Example 6

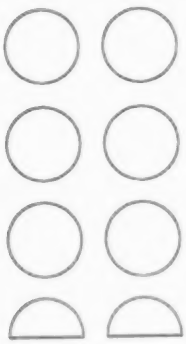
Three dimes are tossed at the same time. What is the probability that exactly two of the coins will be the same?

A. $1/8$
B. $3/8$
C. $3/4$
D. $1/4$

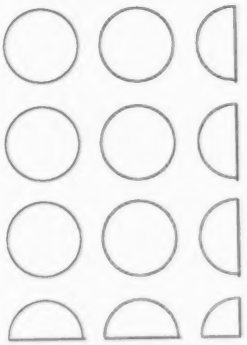
Example 7

Which of these can be shown to correctly represent $2\frac{1}{2} \times 3\frac{1}{2}$?

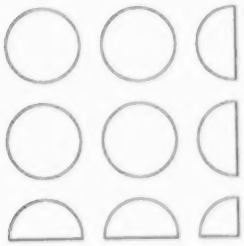
A.



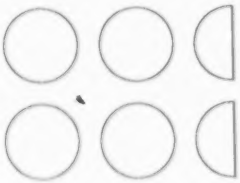
B.



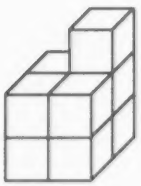
C.



D.



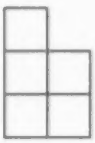
Example 8



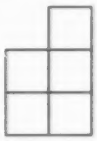
Which of the following is not a top view or side view of the figure above?

A.

C.



B.



D.

Example 9

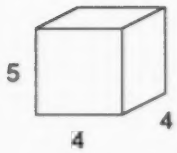
California Averages for High Schools in 1991

• Annual cost per pupil	$3,610
• Number of days in school year	185
• Number of students in average class	24
• Number of classes average student takes each day	5

Based on the information above, which one of these are you NOT able to find?

A. How much a school spends per student for one period of instruction
B. How much a school spends per student over a 5-year period
C. How many students a teacher instructs per day during five class periods
D. How many students a teacher instructs during a year of teaching five periods per day

Example 10

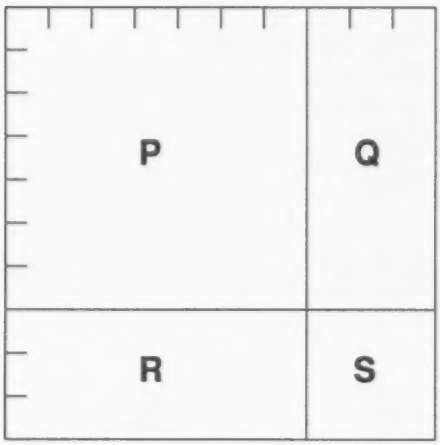


You have been asked to wrap the box above for a birthday present. The best dimensions for a single sheet of wrapping paper, allowing for overlap, are:

A. 5 by 16
B. 6 by 14
C. 9 by 18
D. 10 by 20

Example 11

Which one of the areas in the diagram represents the probability that a basketball player who has a 70% probability of making a basket will make two consecutive baskets?

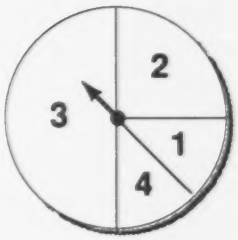


A. Area P
B. Area Q
C. Area R
D. Area S

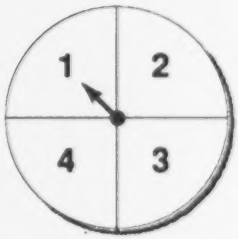
Example 12

Which spinner is most likely to spin a sum greater than seven after two spins?

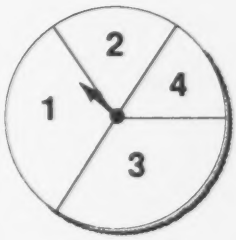
A.



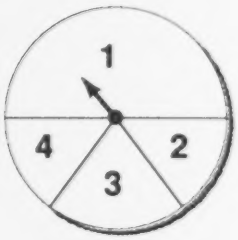
B.



C.

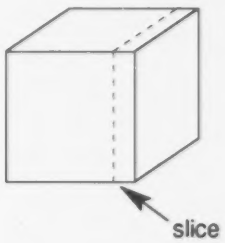


D.



Example 13

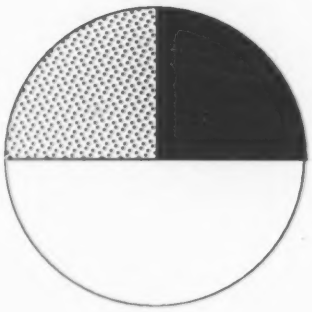
In carpentry class, Pasqual sawed a 1-inch slice from a large wooden cube. The volume of the remaining block was 48 cubic inches.



What was the length in inches of the longest side of the new block?

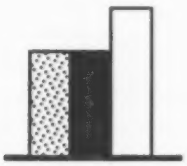
A. 4
B. 8
C. 12
D. 24

Example 14

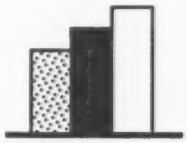


Which one of the following bar graph groups represents the data from the circle graph below?

A.

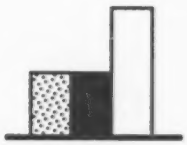


B.



C.

D.



Example 15

Four-inch pancakes are made from a package using dry mix and water. The amounts are shown in the chart:

Pancakes	*Mix*	*Water*
7 - 8	1 cup	$\frac{3}{4}$
14 - 15	2 cups	$1\frac{1}{2}$

If you want to make 30 pancakes, how many cups of water will you need?

A. $2\frac{1}{2}$
B. 3
C. $3\frac{1}{2}$
D. 4

Example 16

Bill is invited to attend a party given today. His invitation reads:

You are invited to a Birthday Party.

Date:	**April 14th**
Time:	**2:30 p.m.**
Place:	**John's House**

Bill has to run an errand for his Mom on the way to the party. It will take him 20 minutes. It takes Bill a half hour to get ready for the party and 15 minutes to get to John's house. Bill's watch reads:

Does Bill have enough time to get to the party on time?

A. Bill will be 10 minutes late.
B. Bill will be 5 minutes late.
C. Bill will have 10 extra minutes to get to the party.
D. Bill will have 20 extra minutes to get to the party.

Selected References

California Department of Education's Publications

For information on ordering the following publications of the California Department of Education, see page 55 of this document.

***The Changing Mathematics Curriculum: A Booklet for Parents*, 1989.**

Also available in Spanish, this booklet introduces parents to today's curricular concepts in mathematics. It tells parents what to expect in their children's classrooms in terms of methodology and teaching aids, and it describes ways in which parents can provide support from home.

***Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve*, 1985.**

Presents a recommended K–12 mathematics program, giving suggestions for both content and structure. Emphasizes problem solving, calculator technology, computational skills, estimation and mental arithmetic, and computers in mathematics education. Gives suggestions for implementing program.

Mathematics Model Curriculum Guide, Kindergarten Through Grade Eight, 1987.

Expands on the *Mathematics Framework* for kindergarten through grade eight, giving more descriptions of what is important for learning content and classroom experiences. Gives sample activities and teaching principles.

***A Question of Thinking, A First Look at Students' Performance on Open-ended Questions in Mathematics*, 1989.**

Reports on results from the first administration of open-ended CAP test items in 1987. For each item a scoring rubric is given, as well as a listing of major student misconceptions with corresponding teaching implications. Also includes sample responses to 1988 open-ended items and nine additional open-ended items.

***Seeing Fractions: A Unit for the Upper Elementary Grades*, 1991.**

Designed to help students interpret and use fractions and to grasp the relationships that fractions represent, this unit was tested in 30 classrooms. It incorporates teachers' feedback from two rounds of trials.

Publications from Other Sources

Assessment Alternatives in Mathematics, An Overview of Assessment Techniques that Promote Learning. **Ed. Jean Kerr Stenmark. Berkeley: EQUALS and the California Mathematics Council, 1989.**

Review of current assessment methods, including portfolios, writing, investigations, open-ended questions, observation, interviews, and self-assessment. The cost is $5 from Assessment Alternatives, c/o EQUALS, Lawrence Hall of Science, University of California, Berkeley, CA 94720.

Clarke, David. ***Assessment Alternatives in Mathematics.*** **The Mathematics Curriculum and Teaching Program, Australia, 1988.**

Australian professional development package for assessment of student learning. The cost is US $25 (VISA and Mastercard accepted) from Publications Officer, CDC, P.O. Box 34, Woden Act 2606, Australia.

ComMuniCator. Journal of the California Mathematics Council (CMC).

Membership in CMC, which includes subscription to *ComMuniCator*, is available for $20 per year. Write to Ruth Hadley, Membership, California Mathematics Council, 1414 S. Wallis, Santa Maria, CA 93454.

Curriculum and Evaluation Standards for School Mathematics. **Reston Vir.: National Council of Teachers of Mathematics, 1989.**

Curriculum standards for grades K-4, 5-8, and 9-12, with a section on evaluation. The cost is $25 from National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091.

Everybody Counts, A Report to the Nation on the Future of Mathematics Education. **Washington D.C.: National Research Council, Mathematical Sciences Education Board, 1989.**

Through a special arrangement with National Academy Press, the California Department of Education is able to provide this publication to California residents at a substantial discount, 30 percent off its regular price: $5 per single copy; $4.25 each for 2–9 copies; $3 each for 10 or more copies. From this document you can learn what educators at the highest level have to say about what must be done to set mathematics education on course to achieve superior results. (See page 56 for information on placing orders.)

Mumme, Judy. ***Portfolio Assessment in Mathematics.*** **California Mathematics Project, Santa Barbara, Calif., 1990.**

Purposes of portfolios, contents and selection of items for portfolios, grading, and benefits of portfolios. Many examples of student work. For information on the document, write to California Mathematics Project, 300 Lakeside Drive, 18th Floor, Oakland, CA 94612-3550.

Publications Available from the Department of Education

This publication is one of over 600 that are available from the California Department of Education. Some of the more recent publications or those most widely used are the following:

ISBN	Title (Date of publication)	Price
0-8011-0722-9	Accounting Procedures for Student Organizations (1988)	$3.75
0-8011-0838-1	Adoption Recommendations of the Curriculum Development and Supplemental Materials Commission to the State Board of Education, 1989: California Basic Instructional Materials in Bilingual Language Arts and Visual and Performing Arts—Visual Arts and Music (1989)	3.50
0-8011-0890-x	Bilingual Education Handbook: A Handbook for Designing Instruction for LEP Students (1990)	4.25
0-8011-0273-1	California Adult Education Handbook (1986)	4.00
0-8011-0687-7	The California CBO: The 1987-88 Profile of Chief Business Officials in California Schools, K–12 (1989)	4.50
0-8011-0862-4	California Education Summit: Background and Final Report (a set) (1990)	5.00
0-8011-0889-6	California Private School Directory (1990)	14.00
0-8011-0924-8	California Public School Directory (1991)	14.00
	California's Daily Food Guide (brochure) (1990)*	6.00/25
0-8011-0874-8	The Changing History–Social Science Curriculum: A Booklet for Parents (1990)†	5.00/10
0-8011-0867-5	The Changing Language Arts Curriculum: A Booklet for Parents (1990)†	5.00/10
0-8011-0928-0	The Changing Language Arts Curriculum: A Booklet for Parents (Spanish Edition) (1991)†	5.00/10
0-8011-0777-6	The Changing Mathematics Curriculum: A Booklet for Parents (1989)†	5.00/10
0-8011-0891-8	The Changing Mathematics Curriculum: A Booklet for Parents (Spanish Edition) (1991)†	5.00/10
0-8011-0907-8	Coordinated Compliance Review Manual, 1990-91 Edition (1990)	6.75
0-8011-0797-0	Desktop Publishing Guidelines (1989)	4.00
0-8011-0833-0	Directory of Microcomputer Software for School Business Administration (1990)	7.50
0-8011-0856-x	English as a Second Language Handbook for Adult Education Instructors (1990)	4.50
0-8011-0041-0	English–Language Arts Framework for California Public Schools (1987)	3.00
0-8011-0927-2	English–Language Arts Model Curriculum Standards: Grades Nine Through Twelve (1991)	4.50
0-8011-0894-2	Enhancing Opportunities for Students Underrepresented in Higher Education (1990)	3.50
0-8011-0786-5	Enrichment Opportunities Guide: A Resource for Teachers and Students in Math and Science (with binder) (1988)‡	8.75
0-8011-0849-7	Food Sanitation and Safety Self-Assessment Instrument for Child Care Centers (1990)	3.75
0-8011-0850-0	Food Sanitation and Safety Self-Assessment Instrument for Family Day Care Homes (1990)	3.75
0-8011-0851-9	Food Sanitation and Safety Self-Assessment Instrument for School Nutrition Programs (1990)	3.75
0-8011-0804-7	Foreign Language Framework for California Public Schools (1989)	5.50
0-8011-0179-4	Handbook for Planning an Effective Mathematics Program, Kindergarten Through Grade Twelve (1982)	2.25
0-8011-0909-4	Handbook on California Education for Language Minority Parents—Portuguese/English Edition (1990)§	3.25
0-8011-0734-2	Here They Come: Ready or Not—Report of the School Readiness Task Force (Full Report) (1988)	4.25
0-8011-0712-1	History–Social Science Framework for California Public Schools (1988)	6.00
0-8011-0782-2	Images: A Workbook for Enhancing Self-esteem and Promoting Career Preparation, Especially for Black Girls (1988)	6.00
0-8011-0828-4	Instructor's Behind-the-Wheel Guide for California's Bus Driver's Training Course (1989)	20.00
0-8011-0358-4	Mathematics Framework for California Public Schools (1985)	3.00
0-8011-0664-8	Mathematics Model Curriculum Guide, K–8 (1987)	2.75
0-8011-0794-6	Microcomputer Software Use in School District Business Offices: Report of a Survey (1989)	4.00
0-8011-0929-9	Model Curriculum Standards, Grades Nine Through Twelve (1985)	5.50
0-8011-0969-8	Not Schools Alone: Guidelines for Schools and Communities to Prevent the Use of Tobacco, Alcohol, and Other Drugs Among Children and Youth (1991)	3.25

*The price for 100 brochures is $16.50; the price for 1,000 brochures is $145.

†The price for 100 booklets is $30; the price for 1,000 booklets is $230. A set of one of each of the parent booklets in English is $3.

‡Also available without binder for $5.75.

§The following editions are also available at the same price: Armenian/English, Cambodian/English, Chinese/English, Hmong/English, Japanese/English, Korean/English, Laotian/English, Pilipino/English, Samoan/English, Spanish/English, and Vietnamese/English.

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ISBN	Title (Date of publication)	Price
0-8011-0306-1	Physical Education for Individuals with Exceptional Needs (1986)	$9.75
0-8011-0834-9	Program Cost Accounting Manual (Form J-380—Form J-580) (1989)	20.00
0-8011-0886-1	Program Guidelines for Individuals Who Are Deaf-Blind (1990)	6.00
0-8011-0817-9	Program Guidelines for Language, Speech, and Hearing Specialists Providing Designated Instruction and Services (1989)	6.00
0-8011-0899-3	Quality Criteria for Elementary Schools: Planning, Implementing, Self-Study, and Program Quality Review (1990)	4.50
0-8011-0906-x	Quality Criteria for High Schools (1990)	4.50
0-8011-0905-1	Quality Criteria for Middle Grades (1990)	4.50
0-8011-0815-2	A Question of Thinking: A First Look at Students' Performance on Open-ended Questions in Mathematics (1989)	6.00
0-8011-0858-6	Readings for Teachers of United States History and Government (1990)	3.25
0-8011-0831-4	Recommended Literature, Grades Nine Through Twelve (1990)	4.50
0-8011-0863-2	Recommended Readings in Literature, Kindergarten Through Grade Eight, Addendum (1990)	2.25
0-8011-0745-8	Recommended Readings in Literature, Kindergarten Through Grade Eight, Annotated Edition (1988)*	4.50
0-8011-0972-8	A Sampler of Mathematics Assessment (1991)	4.00
0-8011-0868-3	School Crime in California for the 1988-89 School Year (1990)	3.50
0-8011-0911-6	Schools for the Twenty-first Century (1990)	3.75
0-8011-0870-5	Science Framework for California Public Schools (1990)	6.50
0-8011-0665-6	Science Model Curriculum Guide, K–8 (1988)	3.25
0-8011-0677-x	Secondary Textbook Review: General Mathematics (1987)	6.50
0-8011-0926-4	Seeing Fractions: A Unit for the Upper Elementary Grades (1991)	7.00
0-8011-0970-1	Self-Assessment Guide for School District Policy Teams: Facilities Planning and Construction (1991)	3.50
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0-8011-0857-8	Self-Assessment Guide for School District Policy Teams: Pupil Transportation Services (1990)	3.50
0-8011-0813-6	Self-Assessment Guide for School District Policy Teams: School Nutrition Program (1989)	3.50
0-8011-0807-1	Statement on Competencies in Languages Other Than English Expected of Entering Freshmen: Phase I—French, German, Spanish (1988)	4.00
0-8011-0836-5	Statement on Competencies in Mathematics Expected of Entering Freshmen (1989)	4.00
0-8011-0855-1	Strengthening the Arts in California Schools: A Design for the Future (1990)	4.50
0-8011-0827-6	Technical Assistance Manual for the California Model School Accountability Report Card (1989)	3.75
0-8011-0846-2	Toward a State of Esteem: The Final Report of the California Task Force to Promote Self-esteem and Personal and Social Responsibility (1990)	4.00
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0-8011-0832-2	Writing Achievement of California Eighth Graders: Year Two (1989)	4.00
0-8011-0887-x	Writing Assessment Handbook, Grade 8 (1990)	8.50

*Includes complimentary copy of *Addendum* (ISBN 0-8011-0863-2).

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A complete list of publications available from the Department, including apprenticeship instructional materials, may be obtained by writing to the address listed above or by calling (916) 445-1260.

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We are using this form to build a roster of mathematics educators who are giving workshops, presentations, seminars, or writing innovative materials. This roster will be used for networking with advisers and mathematics organizations to improve assessment in classrooms. Please check the box below if you want to be placed on this roster, and attach a short description of your activities.

- ❑ Place my name on the CAP workshop presenters' roster.
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